Materials discussed on 4/10

We have discussed the following.

1. Suppose f has essential singularity at 0, then for all $c \in \mathbb{C}, \epsilon > 0$, there exists $\delta, \alpha \in B(c, \epsilon)$ such that $f(z) = \alpha$ has infinity many solutions.

2. Suppose f is of moderate decrease, $f = \sum_{n=0}^{\infty} a_n z^n$ and $\hat{f}$ is compactly supported on $[-M, M]$, then
$$a_n = \frac{(2\pi i)^n}{n!} \int_{-M}^{M} \hat{f}(t) t^n \, dt.$$
Hence, $\limsup [|a_n| n!]^{1/n} \leq 2\pi M$.
Conversely, if the inequality is true, then f is a entire function and $\forall \epsilon > 0$, $\exists A$ such that
$$|f| \leq A e^{2\pi(M+\epsilon)|z|}.$$

3. If f is analytic on $\{z : Im(z) \in [-1, 1]\}$, and $f(x) \in \mathbb{R}$ for all $x \in [0, 1]$. Then $f(x) \in \mathbb{R}$ for all $x \in \mathbb{R}$.

4. (a) Find a entire function so that it has simple zeros precisely at $\log n$ where $n \in \mathbb{N}$.
   (b) Can f constructed at (a) have finite order?
   (c) Find all f of finite order so that $f(\log n) = n$ for all $n \in \mathbb{N}$.

5. Find the Hadamard factorization of $\cosh z - 1$.